COHERENCE ENHANCING DIFFUSION FILTERING BASED ON THE PHASE CONGRUENCY TENSOR

Boguslaw Obara[1,2,*], *Mark Fricker*[3], *Vicente Grau*[1,4]

[1]Oxford e-Research Centre, [2]Oxford Centre for Integrative Systems Biology,
[3]Department of Plant Sciences, [4]Institute of Biomedical Engineering, University of Oxford, UK

ABSTRACT

Many biomedical applications require the enhancement of coherent flow-like curvilinear structures in images. This can be accomplished in a natural way by adopting anisotropic diffusion filtering to local texture analysis by means of the structure tensor. Here we propose a contrast independent anisotropic diffusion filtering of curvilinear structures based on a novel concept: the Phase Congruency Tensor (PCT). The phase angle of the PCT determines the preferred diffusion direction and the diffusity is controlled by PCT-based vesselness measure. We show that the proposed method is largely insensitive to intensity variations along the curve, and provides successful enhancement within noisy regions. The performance of the proposed approach was evaluated by comparing it with a state-of-the-art intensity-based approach on both synthetic and real biological images.

Index Terms— Anisotropic diffusion filtering, Phase Congruency Tensor, coherence enhancing.

1. INTRODUCTION

Most people have an intuitive impression of diffusion as a physical process that equilibrates concentration differences without creating or destroying mass. In image processing we may identify the concentration with the gray value of an image $I(\mathbf{p})$ at a certain location. This observation can be easily cast in a mathematical formulation, the diffusion equation:

$$\frac{\partial I(\mathbf{p})_t}{\partial t} = \nabla \cdot (D \nabla I(\mathbf{p})_t) \qquad (1)$$

where t denotes the diffusion time, D is the diffusion coefficient (which can be a scalar or a tensor), $\mathbf{p} = [x, y]^T$ represents pixel location, $I(\mathbf{p})_t$ is the diffused image at time t, and $I(\mathbf{p})_0 = I(\mathbf{p})$. If $D = 1$, this formula represents a linear diffusion which is exactly the same operation as convolving the image $I(\mathbf{p})_t$ with a Gaussian kernel of variance $\sqrt{2t}$.

A nonlinear variant of the diffusion filter was first suggested in the pioneering work of Perona and Malik [1], where they proposed to replace the scalar diffusion coefficient with a scalar-valued function of the gradient of the intensity levels in the image. The length of the gradient is a good measure of the edge strength of the current location which is dependent on the differential structure of the image. This dependence makes the diffusion process nonlinear, as diffusitivity is rapidly decreasing at edges. This nonlinear process is called anisotropic diffusion filtering as to the fact that the diffusion is performed independently of the orientation of the gradient.

However, in certain cases it is needed that the diffusion is performed in the orientation of interesting features. An anisotropic diffusion filtering that considers the magnitude and direction of the image gradient was introduced by [2]. Such a filtering leads to edge enhancing with noise elimination or coherence enhancing. The coherence enhancing means the enhancement of flow-like curvilinear structures such as fingerprint or tree ring. For these purposes, the filter needs to be perpendicular to edges. A related early coherence enhancing diffusion filtering was introduced by [3]. Thereafter, a tensor driven anisotropic diffusion with scale space analysis was proposed by [4]. This approach was improved by using an optimized rotation invariant scheme in [5] and an iterative scheme in [6]. The use of the image Laplacian instead of the gradient was introduced by [7]. Moreover, [8] have proposed a new coherence measurement based on orientation estimation. However, all these approaches are largely sensitive to intensity variations along the flow-like structures.

In the present paper we propose a contrast independent anisotropic diffusion filtering of flow-like (curvilinear) structures in vector-valued images. This is done by embedding a newly proposed contrast-independent tensor representation, the Phase Congruency Tensor, into the nonlinear diffusion tensor. The phase angle of the PCT regulates the preferred diffusion direction and the diffusity is controlled by a combination of PCT's eigenvalues. We show that in contrast to previous intensity-based methods, the use of local phase makes the proposed method largely insensitive to intensity variations along the curve, and provides successful enhancement within noisy regions. The performance of the proposed approach was evaluated by comparing it with a state-of-the-art intensity-based approach on both synthetic and real biological images.

This work was supported by grants from the BBSRC, EPSRC and NERC, UK. *boguslaw.obara@oerc.ox.ac.uk

1.1. Anisotropic Diffusion

Anisotropic diffusion filtering is defined as in Equation 1 by exchanging the scalar diffusion coefficient D with a positive definite symmetric matrix which represents the diffusion in both intensity and orientation [3]. The tensor D can be built to control the direction and intensity of the diffusion at each point. In [4], for example, the use of the structure tensor in proposed. Such a structure tensor J is defined as [9]:

$$J(\nabla I(\mathbf{p})_\sigma)_\rho = G_\rho * \left(\nabla I(\mathbf{p})_\sigma \nabla I(\mathbf{p})_\sigma^T\right) = \begin{bmatrix} J_{11} & J_{12} \\ J_{11} & J_{22} \end{bmatrix} \quad (2)$$

where the function G_ρ denotes a Gaussian with a standard deviation ρ, and $I(\mathbf{p})_\sigma = G_\sigma * I(\mathbf{p})$ is a regularized version of $I(\mathbf{p})$ that is obtained by convolution with a Gaussian G_σ. Using eigendecomposition, $J(\mathbf{p})_\sigma$ can be expressed as follows:

$$J(\nabla I(\mathbf{p})_\sigma)_\rho = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix} \quad (3)$$

with $\mu_1 > \mu_2$. The diffusion tensor D must steer a filtering process such that the diffusion is strong mainly along the coherence direction defined by $v_1 = [v_x, v_y]$, the eigenvector corresponding to the eigenvalue of largest magnitude, and it increases with the coherence, which in [4] is defined as $(\mu_1 - \mu_2)^2$. To obtain that, D is defined as follows:

$$D = \begin{bmatrix} v_x & -v_y \\ v_y & v_x \end{bmatrix} \begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{bmatrix} \begin{bmatrix} v_x & -v_y \\ v_y & v_x \end{bmatrix}^{-1} \quad (4)$$

$$\kappa_1 = c_1$$
$$\kappa_2 = \begin{cases} c_1 & \text{if } \mu_1 = \mu_2, \\ c_1 + (1 - c_1) e^{\left(\frac{-c_2}{(\mu_1 - \mu_2)^2}\right)} & \text{else} \end{cases} \quad (5)$$

where $\kappa_{1,2}$ are the eigenvalues of D, $c_1 \in (0, 1)$, $c_1 \ll 1$, and $c_2 > 0$.

2. METHOD

The local structure of an image, in particular if it is formed by piecewise curvilinear segments, can be described by analyzing the relations between eigenvalues and eigenvectors of the locally-calculated gradient-based structure tensor, see Equation 2. In a similar way, we propose to deduct the orientation of a curvilinear structure by the eigenvalues and eigenvectors of a novel phase concept, called the Phase Congruency Tensor (PCT) [10], bringing the advantage of contrast invariance offered by a phase-based approach. Thus, we propose to embed PCT into the nonlinear diffusion tensor D in the way that D steers a filtering process such that the diffusion is strong mainly along the coherence direction defined by PCT eigenvectors, and it increases with the coherence given by PCT-based vesselness.

2.1. Phase Congruency Tensor

In general, local tensor-based representations can be produced by combining the outputs from polar separable quadrature filters, applied on several orientations.While tensor representations can be built on purely intensity-based filters, these have the downside of being sensitive to changes in image contrast. Methods based on local phase have been proposed as a contrast-independent alternative for feature detection. In particular, phase congruency [11] is based on the concept that salient features have similar values of local phase when observed at different scales. Here we exploit the idea that phase congruency values are high in the direction perpendicular to the structure, while they remain close to zero in the direction parallel to the structure. More importantly, the values of phase congruency are minimally affected by contrast changes. Thus, for a given set of scales $\{s\}$, a set of orientations $\{o\}$ and a given set of phase congruency measures $PC_o(\mathbf{p})$ (for each orientation o) [11], the Phase Congruency Tensor takes the following form [10]:

$$T_{PC} = \sum_o PC_o(\mathbf{p})(\mathbf{n}_o \mathbf{n}_o^T - \alpha \mathbb{I}) \quad (6)$$

where $\mathbf{n}_o$ is the normalized orientation vector in the direction o, $\alpha = \frac{1}{m-1}$, with m being the dimensionality of the image and $\mathbb{I}$ is the identity tensor.

The eigen-decomposition of the tensor T_{PC} results in $\lambda_{1,2}$ and $t_{1,2}$ for the eigenvalues and eigenvectors respectively.

2.2. PCT Vesselness

Piecewise curvilinear segments can be detected by analyzing the relations between eigenvalues and eigenvectors of the locally-calculated Hessian.In a similar way, the dominant orientation of the surface representing a curvilinear structure is given by the dominant eigenvector of T_{PC}, with the advantage of contrast-invariance. In [12], a measure of coherence of curvilinear structure named vesselness was introduced. Here we propose the use of PCT-based vesselness, calculated using the following formula [10]:

$$V_{PC} = e^{-\frac{\lambda_1^2}{2\beta^2 \lambda_2^2}} \left(1 - e^{-\frac{\lambda_1^2 + \lambda_2^2}{2c^2}}\right) \quad (7)$$

where β and c are thresholds which control the sensitivity of the line measurement.

2.3. PCT Anisotropic Diffusion

Let us consider the tensor T_{PC}, its eigenvalues $\lambda_{1,2}$ and eigenvectors $t_{1,2}$, and the corresponding PCT vesselness V_{PC}. To make the structure descriptor invariant under sign changes, we replace a vector field defined by eigenvectors $t_{1,2}$, by its tensor product defined as follows:

$$S_{PC} = G_\rho * \begin{bmatrix} u_1^2 & u_1 u_2 \\ u_1 u_2 & u_2^2 \end{bmatrix} \quad (8)$$

where $u_1 = -V_{PC} t_y$ and $u_2 = V_{PC} t_x$, $[t_x, t_y]$ is the eigenvector corresponding to the eigenvalue of largest magnitude. Such defined structure tensor S_{PC} is then decomposed into its constituent eigenvectors and corresponding eigenvalues. Finally, a new PCT-based nonlinear diffusion tensor D_{PC} is constructed as described in Equation 4. Its unitary matrices are defined by the eigenvectors of S_{PC} and its diagonal matrix is given by eigenvalues defined similarly to [4]:

$$\kappa_1 = c_1$$
$$\kappa_2 = \begin{cases} c_1 & \text{if } \lambda_1 = \lambda_2, \\ c_1 + (1 - c_1)e^{\left(\frac{-c_2}{V_{PC}}\right)} & \text{else} \end{cases} \tag{9}$$

2.4. Discretization Of Anisotropic Diffusion Equation

For the proposed PCT-based diffusion tensor D_{PC}, the anisotropic diffusion Equation 1 is solved using the finite difference method. The spatial derivatives are approximated by central differences, while the temporal derivative is replaced by its forward difference approximation. This is done essentially by a convolution of the image with a mask with weights varying both spatially and temporally. The spatial discretization used in our work results in 3x3 stencils and corresponds to the non-negativity discretization [5].

3. RESULTS

In order to test the performance of the proposed approach, we applied the coherence enhancing method, based on the description in Section 2, to the enhancement of flow-like curvilinear structures in synthetic images (see Figure 1) and real biological images of saprotrophic fungal and retinal vascular networks (see Figures 2 and 3). Results presented in Figures 1 - 3 were obtained by applying the anisotropic diffusion filtering method with the parameter $c_2 = 0.9$ and the number of iterations $t = 60$. In addition, a contrast-to-noise ratio metric, for grid lines in synthetic image with an increasing noise level, is presented in Table 1. The obtained results highlight the improvement obtained by the phase-based approach when applied on structures with different contrast.

Noise Variance	Image	$(\mu_1 - \mu_2)^2$	V_{PC}
0.1	39.31	48.74	55.38
0.2	30.84	46.88	55.30
0.3	26.58	46.67	52.51
0.4	22.79	45.66	51.96

Table 1. Contrast-to-noise ratio for grid lines in synthetic images with 4 different noise levels and in their corresponding coherence enhanced versions based on $(\mu_1 - \mu_2)^2$ and V_{PC}.

4. DISCUSSION AND FUTURE WORK

A novel nonlinear diffusion tensor constructed by PCT eigenvalues and eigenvectors has been introduced in Section 2. In Section 3, the anisotropic diffusion filtering concept using the proposed PCT-based diffusion tensor, was compared with its corresponding intensity-based version (as defined in Section 1.1). The results shown in Figure 1 illustrate the large variations in output for the intensity-based diffusion tensor approach, when applied to a synthetic image of curvilinear structures with varying contrast. This is due to the fact that the structure tensor J as well as its eigenvalues $(\mu_{1,2})$ and eigenvectors $(v_{1,2})$, used to construct the diffusion tensor D, are intensity dependent (see Equation 2 and Figures 1(b), and 1(d)).

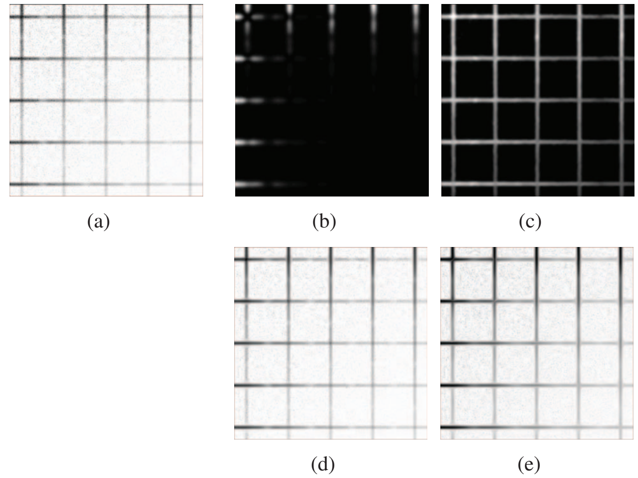

Fig. 1. Comparison between feature enhancement methods on synthetic images. (a) original synthetic image showing branching patterns, (b,c) $(\mu_1 - \mu_2)^2$ and V_{PC} shown to ilustrate variation in diffusity, (d,e) enhancement results obtained by intensity- and our PCT-based approach.

This strongly affects the determination of the preferred diffusion direction and the magnitude of the diffusity supervised by $\kappa_{1,2}$. Moreover, this complicates the choice of a single "threshold" value c_2 that would be suitable to effectively control the coherence enhancement of curvilinear features varying in contrast. On the contrary, results presented for the PCT-based approach show a much higher degree of uniformity in the diffusity, which greatly facilitates accurate feature enhancement, see Figures 1(c), and 1(e). It is apparent from the Figures that PCT-based methods are the preferred option when structures with different contrast are present.

To evaluate the performance of the new approach, we applied a PCT-based coherence filtering method for fungal and vascular network enhancement, and the results are presented in these Figures 2 and 3. Results presented in Figures highlight the improvement obtained by the phase-based approach when applied on structures with different contrast and noise. Please note that Hessian-based vesselness measurement is not valid at the branching and end points due to the ambiguity of the directions of the Hessian eigenvectors. Threfore, the PCT vesselness suffers from these issues too.

To summarize, the obtained results show that PCT-based anisotropic diffusion filtering is robust against changes of intensity contrast of curvilinear structures and capable of providing high enhancement responses on low contrast edges. These properties are essential for detecting the structures in low contrast regions of images, which can contain intensity inhomogeneity; these structures are common in a large number of biomedical images.

The PCT, through the use of its eigenvalues and eigenvectors, can be used to reduce contrast dependency in many approaches for detection and analysis of curvilinear structures observed in biomedical images [10]. Its use in anisotropic

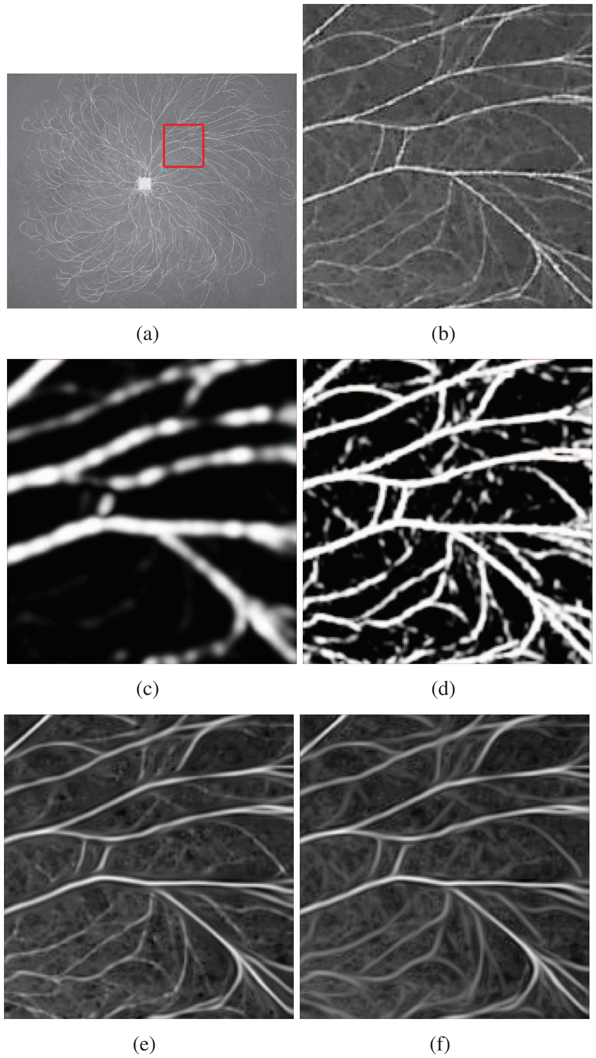

Fig. 2. Comparison between feature enhancement methods on fungal network images. (a) original image showing branching patterns, (b) region of interest, (c,d) $(\mu_1 - \mu_2)^2$ and V_{PC} shown to ilustrate variation in diffusity, (e,f) enhancement results obtained by intensity- and our PCT-based approach.

diffusion filtering has been illustrated in this paper.

5. REFERENCES

[1] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE T. Pattern Anal.*, vol. 12, no. 7, pp. 629–639, 1990.

[2] M. Nitzberg and T. Shiota, "Nonlinear image filtering with edge and corner enhancement," *IEEE T. Pattern Anal.*, vol. 14, no. 8, pp. 826–833, 1992.

[3] J. Weickert, "Theoretical foundations of anisotropic diffusion in image processing," *Comp. Suppl.*, vol. 11, no. 7, pp. 221–236, 1996.

[4] J. Weickert, "Coherence-enhancing diffusion filtering," *Int. J. Comput. Vision*, vol. 31, no. 2-3, pp. 111–127, 1999.

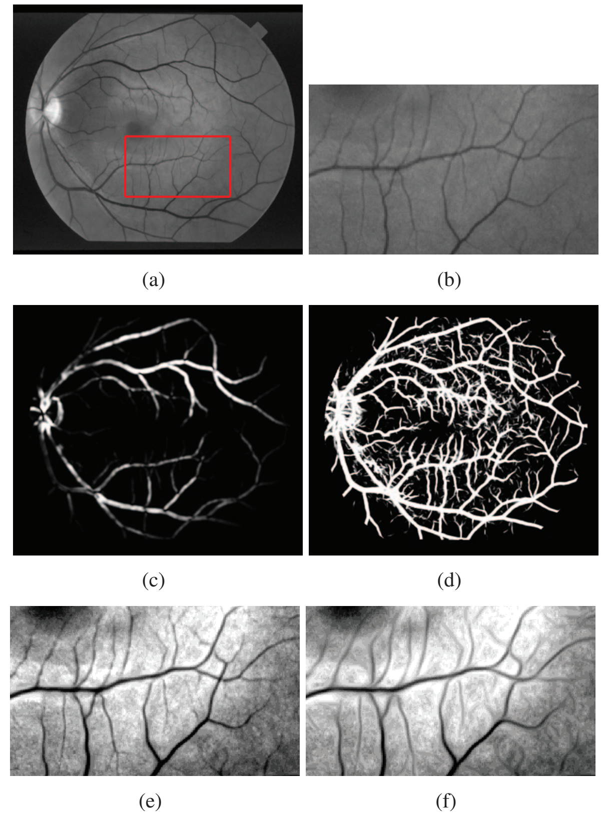

Fig. 3. Comparison between feature enhancement methods on retinal vascular network images. (a) a sample retina image from STARE database, (b) region of interest, (c,d) $(\mu_1 - \mu_2)^2$ and V_{PC} shown to ilustrate variation in diffusity, (e,f) enhancement results obtained by intensity- and our PCT-based approach.

[5] J. Weickert and H. Scharrb, "A scheme for coherence-enhancing diffusion filtering with optimized rotation invariance," *J. Vis. Commun. Image R.*, vol. 13, no. 1-2, pp. 103–118, 2002.

[6] W. Wang, "Generalized explicit schemes for coherence enhancing diffusion filtering," in *ICIG*, 2007, vol. 3708, pp. 126–132.

[7] H. Nishiguchi et al., "Multiscale texture enhancement," in *ICPR*, 2006, vol. 3, pp. 838–841.

[8] R. Terebes et al., "Flow coherence diffusion. Linear and nonlinear case," in *ACIVS*, 2005, vol. 3708, pp. 316–323.

[9] F. Catte et al., "Image selective smoothing and edge detection by nonlinear diffusion," *SIAM J. Numer. Anal.*, vol. 29, no. 1, pp. 182–193, 1992.

[10] B. Obara et al., "Contrast independent detection of branching points in network-like structures," in *SPIE Med. Imaging*, San Diego, California, USA, 2012.

[11] Peter Kovesi, "Phase congruency: A low-level image invariant," *Psychological Research*, vol. 64, no. 2, pp. 136–148, 2000.

[12] A.F. Frangi et al., "Multiscale vessel enhancement filtering," in *MICCAI*, 1998, vol. 1496, pp. 130–137.